Table of Contents

SIO221A Homework #3 Rev3 (Eric Gallimore)

```
function hw3_3()
```

1) Forming a spectral estimate of Vertical Displacement:

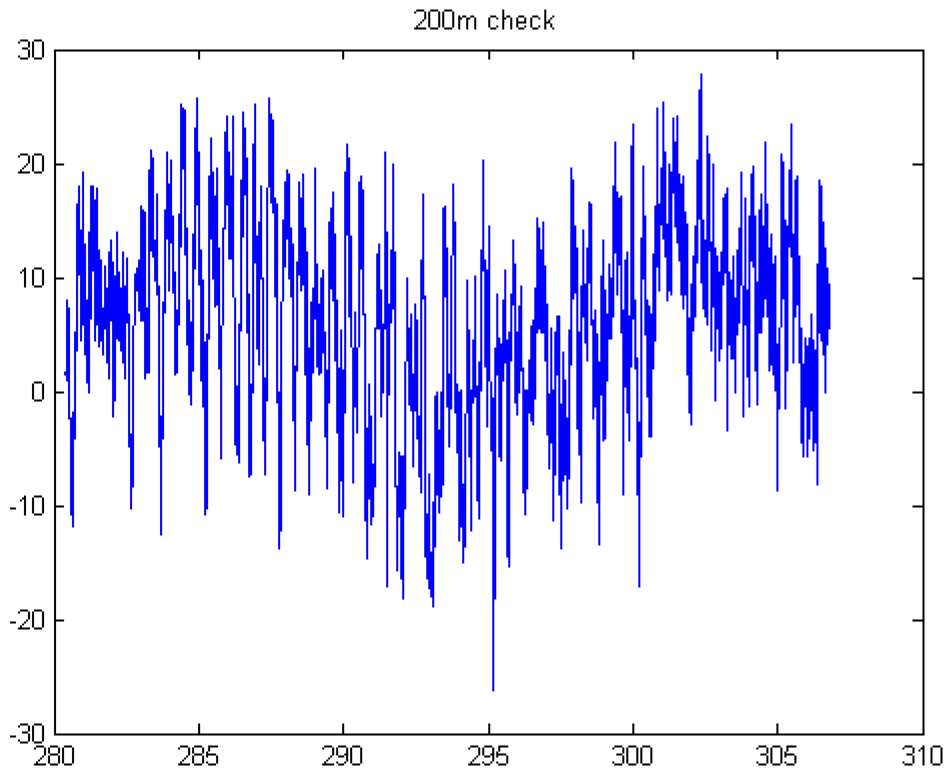
Produce estimates of the power spectrum of isopycnal vertical displacement using isopycnals whose mean depths are 200 m and 500 m.

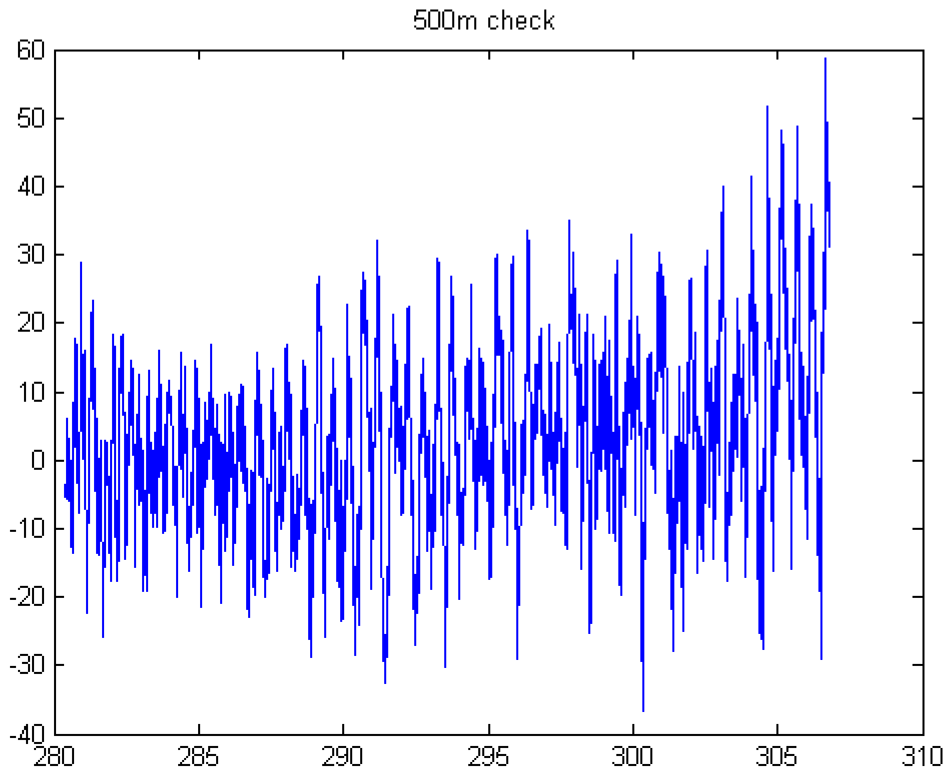
```
clear();
% OK, first, load the data
load('farfield_Disp_prelim.mat')

% Now, find the indexes of the 200m and 500m isopycnals.
index_200m = find(depth >= 200, 1);
index_500m = find(depth >= 500, 1);

%   Here’s how:
%
%
```

1. Plot the data and check for “glitches”. This is always the first step.

```
% Plot the 200m case.  This is just a quick check, so we aren't producing a
% publication-worthy plot.
figure(1);
clf();
plot(time, d(index_200m, :));
title('200m check');

% and repeat for the 500m case.
figure(2);
```

```
clf();
plot(time, d(index_500m, :));
title('500m check');

% Visually, those both look good, so we will proceed.
```





2. Choose a clean segment of daata a least 6000 points long.

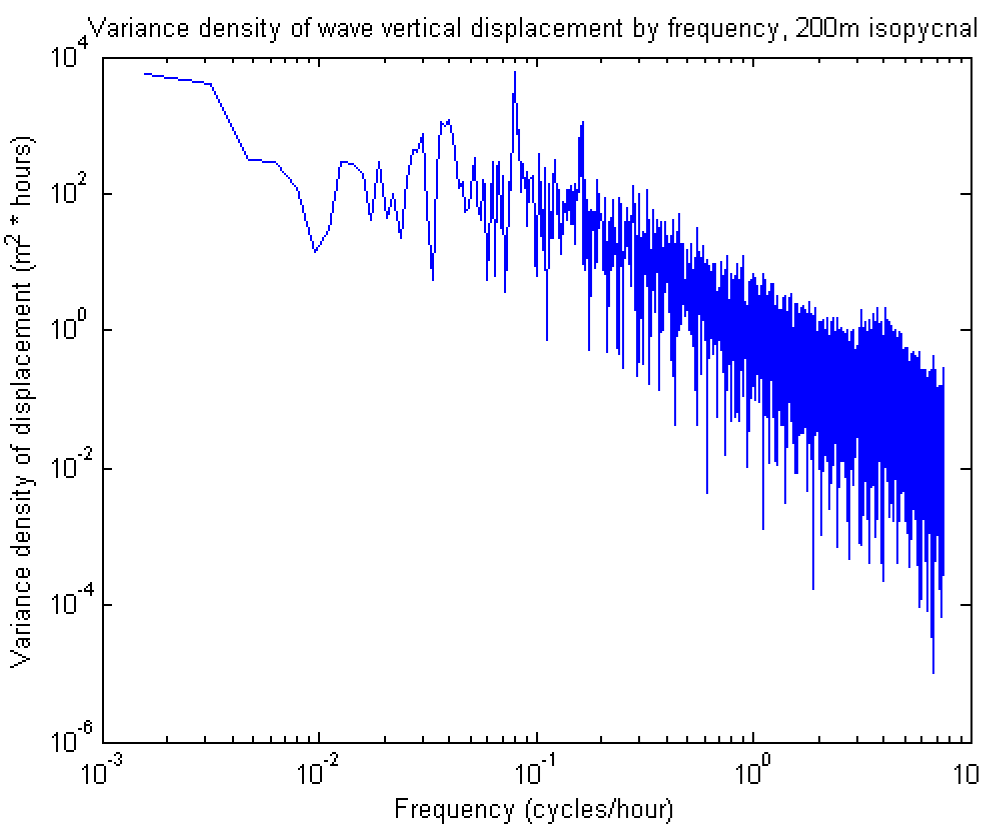
(?t = 4 min = 1/15 hr) If there are “glitches”, set them equal to the mean value of the record. (“? =mean(D(100,:))”)

```
data_200m = d(index_200m, :);
data_500m = d(index_500m, :);

data_interval_secs = 4*60;
data_interval_hours = data_interval_secs / 60 / 60; % = 1/15

%  3. Fourier transform and form the spectral estimate
%  Here, N points have been Fourier transformed, the record duration is N /15 hour
%
%  Note 1: For a real time-series, the spectrum will be mirror symmetric about “ze
%
%  Note 2: The Fourier transform routine in Matlab is  normalized such that you ne
%
```

200m case: Fourier transform and form the spectral estimate

```
[ehat_200m, freq_bins_200m, delta_f_200m] = ...
    varspec_est_real(data_200m, data_interval_hours);

%  Verify that sum(fourier coefficents) = sample variance
variance_200m = var(data_200m);
variance_200m_in_fft = sum(ehat_200m*delta_f_200m);
fprintf('Variance of 200m data (using var): %d m^2\n', variance_200m);
fprintf('Variance of 200m data (using fft): %d m^2\n', variance_200m_in_fft);

%  Plot log10(E(f)   vs. log10 (f) in cycles per hour. Label All axes.
figure(3);
clf();
loglog(freq_bins_200m, ehat_200m);
ylabel('Variance density of displacement (m^2 * hours)')
xlabel('Frequency (cycles/hour)');
title('Variance density of wave vertical displacement by frequency, 200m isopycnal
```

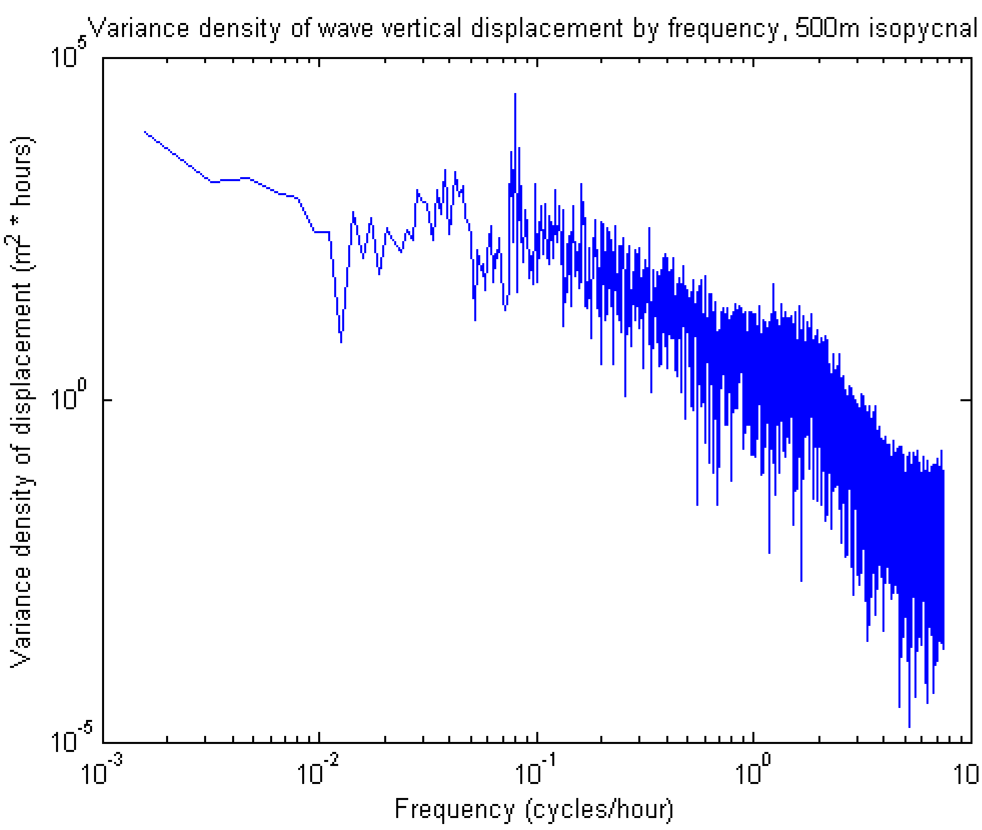
```
Variance of 200m data (using var): 6.948121e+001 m^2

Variance of 200m data (using fft): 6.947400e+001 m^2
```



500m case: Fourier transform and form the spectral estimate

```
[ehat_500m, freq_bins_500m, delta_f_500m] = ...
    varspec_est_real(data_500m, data_interval_hours);

%  Verify that sum(fourier coefficents) = sample variance
variance_500m = var(data_500m);
variance_500m_in_fft = sum(ehat_500m*delta_f_500m);
fprintf('Variance of 500m data (using var): %d m^2\n', variance_500m);
fprintf('Variance of 500m data (using fft): %d m^2\n', variance_500m_in_fft);

%Plot log10(E(f)   vs. log10 (f) in cycles per hour. Label All axes.
figure(4);
clf();
loglog(freq_bins_500m, ehat_500m);
ylabel('Variance density of displacement (m^2 * hours)')
xlabel('Frequency (cycles/hour)');
title('Variance density of wave vertical displacement by frequency, 500m isopycnal
```

```
Variance of 500m data (using var): 1.669780e+002 m^2

Variance of 500m data (using fft): 1.669604e+002 m^2
```

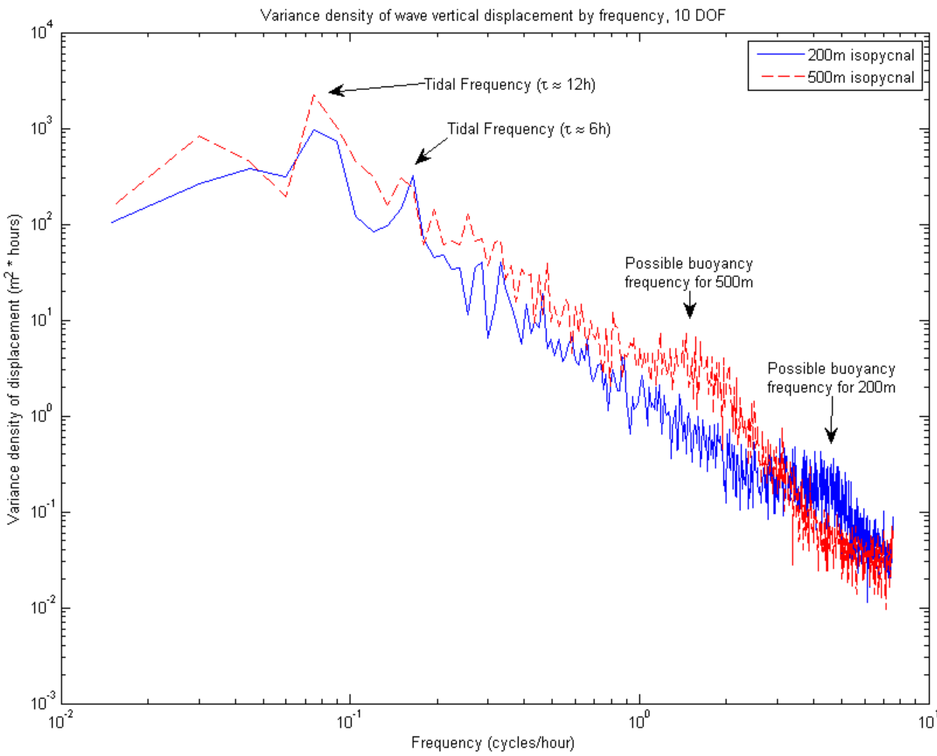


2) Increasing statistical stability:

Now, repeat “problem 1” six times, forming “averaged” spectral estimates at 200m and 500 m using times 1-1000,1001-2000,…5001-6000. Average each set of estimates together at like values of frequency, normalize to preserve variance, and plot log-log vs. frequency. Put the spectral estimates from the 200m and 500m depth zones on the same plot. Identify the tidal and buoyancy frequencies. Note a few other differences/similarities in the data.

```
% 200m case
[Ehat_200m_average, freq_bins_200m_average, delta_f_200m_average] = ...
    varspec_est_avg(data_200m, data_interval_hours, 1000, 5);

% 500m case
[Ehat_500m_average, freq_bins_500m_average, delta_f_500m_average] = ...
    varspec_est_avg(data_500m, data_interval_hours, 1000, 5);

% Plot these values.
figure(5);
clf();

loglog(freq_bins_200m_average, Ehat_200m_average);
hold on;
loglog(freq_bins_500m_average, Ehat_500m_average, 'r--');
ylabel('Variance density of displacement (m^2 * hours)')
xlabel('Frequency (cycles/hour)');
legend('200m isopycnal', '500m isopycnal');
title('Variance density of wave vertical displacement by frequency, 10 DOF');

% Identify the tidal frequencies (identified because we expect something
% with a period of about 6 hours and something with a period of about 12
% hours).
annotation(gcf(),'textarrow',[0.454818122033091 0.369134515119917],...
    [0.86242774566474 0.852272727272727],'TextEdgeColor','none',...
    'String',{'Tidal Frequency (\tau \approx 12h)'});
annotation(gcf(),'textarrow',[0.474452554744526 0.446298227320126],...
    [0.795454545454546 0.765625],'TextEdgeColor','none',...
    'String',{'Tidal Frequency (\tau \approx 6h)'});

% The buoyancy frequency should be somewhere around 0.5-5 cph.  I don't see
% anything obvious here, but there are some possibilities.
annotation(gcf(),'textarrow',[0.691883559463435 0.692321744126972],...
    [0.610044344737742 0.577327581731963],'TextEdgeColor','none',...
    'String',{'Possible buoyancy','frequency for 500m'});
annotation(gcf(),'textarrow',[0.818561001042753 0.817876217161742],...
    [0.481534090909091 0.42478090259997],'TextEdgeColor','none',...
    'String',{'Possible buoyancy','frequency for 200m'});

disp('Note a few other differences/similarities in the data:');
disp('In both cases, the variance is higher at lower frequencies.');
disp('The variance is spread more in the 500m data.  If we have correctly');
disp('identified the buoyancy frequency, this may suggest that the density');
disp('stratification at 500m is less stable');
```

```
Note a few other differences/similarities in the data:

In both cases, the variance is higher at lower frequencies.
The variance is spread more in the 500m data.  If we have correctly
identified the buoyancy frequency, this may suggest that the density
stratification at 500m is less stable
```



3) Forming a spectral estimate of Vertical Velocity:

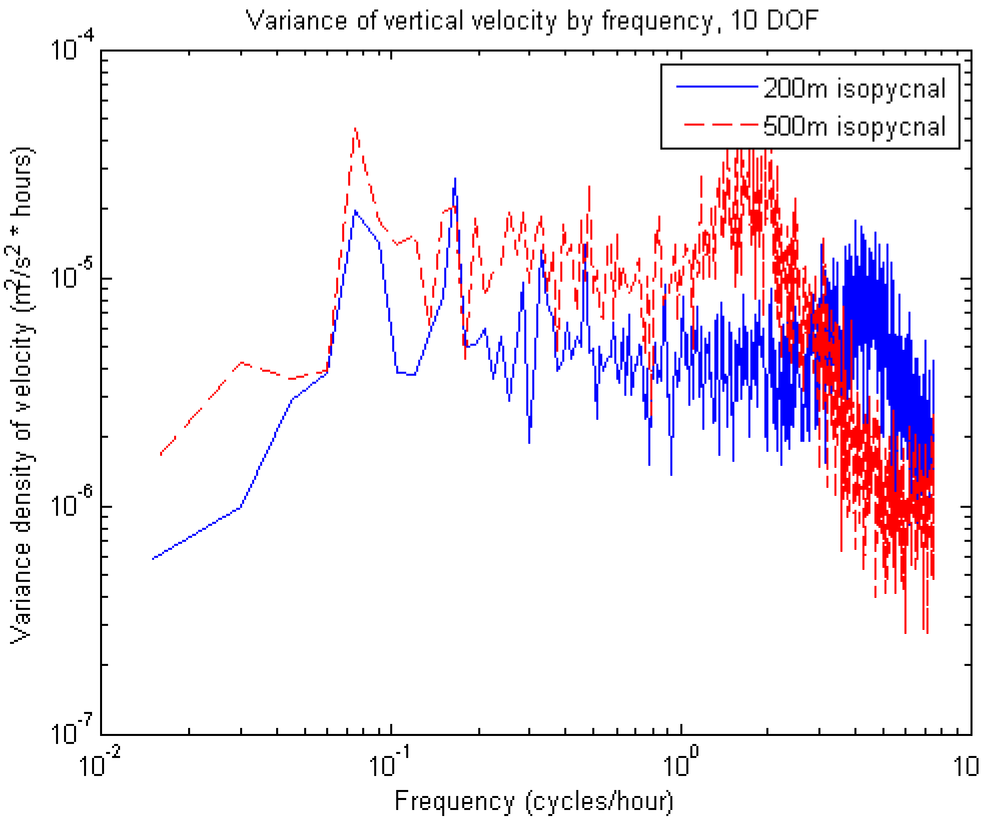
Repeat problem 2 using a series of "effective vertical velocity" Normalize correctly, use frequency in cycles per hour, velocity in meters/s. - Plot log-log, positive frequencies only. State the units, label the axes. - Discuss the physical implications of the similarity/difference between spectral estimates at the two depths.

```
% Make a time-series of velocity for 200m data
% 240 seconds between samples
w_200m = (data_200m(1:end-1) - data_200m(2:end)) ./ 240; % m/s

% And do it for 500m
w_500m = (data_500m(1:end-1) - data_500m(2:end)) ./ 240; % m/s

% Optionally look at the time series of this data to do a sanity check,
% look for glitches.
if (0)
    figure()
```

```matlab
    plot(w_200m);
    figure()
    plot(w_500m);
end
% There are a few outliers in the 200m data, but they don't look bad enough
% to manually correct.


% Get the average values
[Ehat_w_200m_average, freq_bins_w_200m_average, delta_f_w_200m_average] = ...
    varspec_est_avg(w_200m, data_interval_hours, 1000, 5);
[Ehat_w_500m_average, freq_bins_w_500m_average, delta_f_w_500m_average] = ...
    varspec_est_avg(w_500m, data_interval_hours, 1000, 5);

% Plot these values.
figure(6);
clf();

loglog(freq_bins_w_200m_average, Ehat_w_200m_average);
hold on;
loglog(freq_bins_w_500m_average, Ehat_w_500m_average, 'r--');
% The units on this are a bit confusing.  I think that this is the clearest
% way to represent them.
ylabel('Variance density of velocity (m^2/s^2 * hours)')
xlabel('Frequency (cycles/hour)');
legend('200m isopycnal', '500m isopycnal');
title('Variance of vertical velocity by frequency, 10 DOF');


% Check that we are on the right track.
% Verify that sum(fourier coefficents) = sample variance
% The 500m numbers look a bit off, but at least they are the same OOM.
variance_w_200m = var(w_200m(1:6000));
variance_w_200m_in_fft = sum(Ehat_w_200m_average*delta_f_w_200m_average);
fprintf('Variance of 200m velocity data (using var): %d m^2/s\n', ...
    variance_w_200m);
fprintf('Variance of 200m velocity data (using fft): %d m^2/s\n', ...
    variance_w_200m_in_fft)
fprintf('Mean of 200m velocity data (using mean & abs): %d m/s\n', ...
    mean(abs(w_200m(1:6000))));

variance_w_500m = var(w_500m(1:6000));
variance_w_500m_in_fft = sum(Ehat_w_500m_average*delta_f_w_500m_average);
fprintf('Variance of 500m velocity data (using var): %d m^2/s\n', ...
    variance_w_500m);
fprintf('Variance of 500m velocity data (using fft): %d m^2/s\n', ...
    variance_w_500m_in_fft);
fprintf('Mean of 500m velocity data (using mean & abs): %d m/s\n', ...
    mean(abs(w_500m(1:6000))));


% - Discuss the physical implications of the similarity/difference between
%   spectral estimates at the two depths.
disp('The vertical velocity at 500m is greater than the velocity at');
```

```
disp('200m (from mean).');
disp('The velocity at 500m is changing more, particularly at');
disp('low frequencies.  This suggests that this depth is less stable.');
disp('Perhaps the density as a function of depth is more variable around');
disp('500m, thus causing the buoyancy frequency and the amplitude of the');
disp('related wave components to vary more, which matches our expectation');
disp('from the displacement data.');
disp('There is some evidence of a tidal-period related change in velocity');
disp('at both isopycnals.');
disp('(Examining the time series data supports these conclusions.)');
```

```
Variance of 200m velocity data (using var): 3.784630e-005 m^2/s
Variance of 200m velocity data (using fft): 3.704667e-005 m^2/s
Mean of 200m velocity data (using mean & abs): 4.575273e-003 m/s
Variance of 500m velocity data (using var): 5.091792e-005 m^2/s
Variance of 500m velocity data (using fft): 5.006081e-005 m^2/s
Mean of 500m velocity data (using mean & abs): 5.585836e-003 m/s
The vertical velocity at 500m is greater than the velocity at
200m (from mean).
The velocity at 500m is changing more, particularly at
low frequencies.  This suggests that this depth is less stable.
Perhaps the density as a function of depth is more variable around
500m, thus causing the buoyancy frequency and the amplitude of the
related wave components to vary more, which matches our expectation
from the displacement data.
There is some evidence of a tidal-period related change in velocity
at both isopycnals.
(Examining the time series data supports these conclusions.)
```



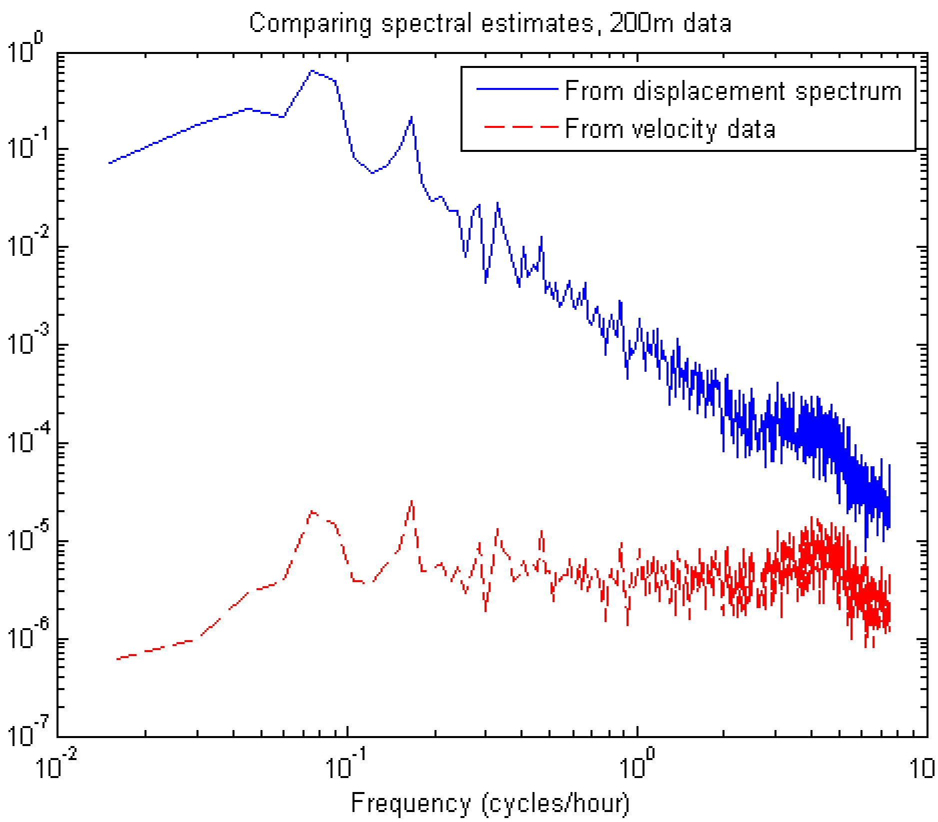
4) Comparing Spectral Estimates

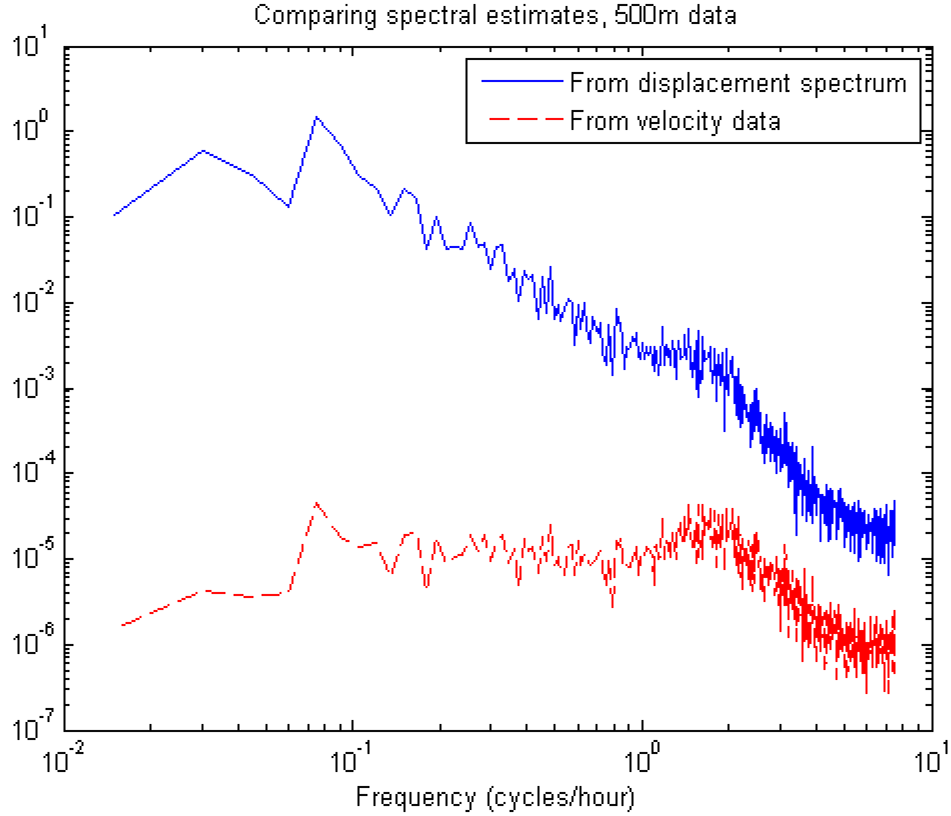
Take the displacement spectra of problem 2 and multiply them by (2*pi*frequency)^2, where the frequency is in cycles per second. How do these spectra compare with those in part 3. (Plot them in the same plot). Why should these different spectral estimates compare?

```
f_Hz = 1/(4*60); %Hz

% 200m case
figure(7);
clf();
loglog(freq_bins_200m_average, ...
    Ehat_200m_average(1:length(freq_bins_200m_average)) .* ((2*pi*f_Hz)^2));
hold on;
loglog(freq_bins_w_200m_average, Ehat_w_200m_average, 'r--');
% I don't think we have a single axis label here.
%ylabel('Variance of velocity (m^2/s * hours)')
xlabel('Frequency (cycles/hour)');
legend('From displacement spectrum','From velocity data');
title('Comparing spectral estimates, 200m data');

% 500m case
figure(8);
clf();
loglog(freq_bins_500m_average, ...
    Ehat_500m_average(1:length(freq_bins_500m_average)) .* ((2*pi*f_Hz)^2));
```

```
hold on;
loglog(freq_bins_w_500m_average, Ehat_w_500m_average, 'r--');
% I don't think we have a single axis label here.
%ylabel('Variance of velocity (m^2/s * hours)')
xlabel('Frequency (cycles/hour)');
legend('From displacement spectrum','From velocity data');
title('Comparing spectral estimates, 500m data');

disp('These spectral estimates compare because the velocity and displacement');
disp('are related by a factor of time: velocity is the time derivative of');
disp('displacement.');
```

```
These spectral estimates compare because the velocity and displacement

are related by a factor of time: velocity is the time derivative of
displacement.
```






```
end

% Take the given data and provide a variance density estimate
% The freq_bins array will be in reciprocal units to the units of
% sample_interval.
% ehat has units of (data_units^2) per (1/sample_interval_units)
function [ehat, freq_bins, delta_f] = ...
    varspec_est_real(data, sample_interval)

    % First, do the FFT to get an array of Fourier coefficients
    a = fft(data);

    % Figure out our frequency bins.
    N = length(data);
    delta_f = (1/sample_interval) / N;  % same as fs / N
    freq_bins = 0:delta_f:(N/2 * delta_f);

    % We are dealing with a real signal, so we only care about positive, real
    % freqency components.  To get the power, we want to "fold" both sides of
    % the spectrum together.  Since they are the same, we can just take one
    % side and multiply by 2.
    % To preserve variance, we have to normalize this result by dividing by
    % N^2, due to the Matlab implementation of fft.
    ehat = 2 * abs(a(1:length(freq_bins))).^2 ./ delta_f ./ ...
        (N^2);
```

```
    % The first frequency bin will hold the mean, not a component of the
    % variance.  Since we are interested in the variance spectrum, drop
    % the first bin.
    ehat = ehat(2:end);
    freq_bins = freq_bins(2:end);

end

function [ehat, freq_bins, delta_f] = ...
    varspec_est_avg(data, sample_interval, set_size, number_of_sets)

    % Get the variance spectral density for each subset.
    for i=1:number_of_sets
        start_idx = (i-1) * set_size + 1;
        end_idx = start_idx + set_size - 1;

        [ehat_parts(i, :), freq_bins, delta_f] = ...
            varspec_est_real(data(start_idx:end_idx), sample_interval);
    end

    % form the average
    % Get the average values (average of columns)
    ehat = mean(ehat_parts);

end
```

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