
SIO221A Homework #7 (Eric Gallimore)

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Initialization

```
function hw7()

clear();
close all;
load('velb4degl.mat');

% Figure out how to partition this to get 50 DOF
dof = 50;
len_complete = size(velb4, 2);
points_per = floor(len_complete / dof);
```

Step 1

Estimate $H(\sigma)$ between ranges 100 and 90 using spectra and cross spectral estimates calculated at 50 dof, as per Homework 6. Plot the Gain and Phase functions at positive and negative frequencies.

```
sample_interval = 0.625; % seconds

velocity_100 = reshape(velb4(100,:), [], 1);
velocity_90 = reshape(velb4(90,:), [], 1);
velocity_80 = reshape(velb4(80,:), [], 1);

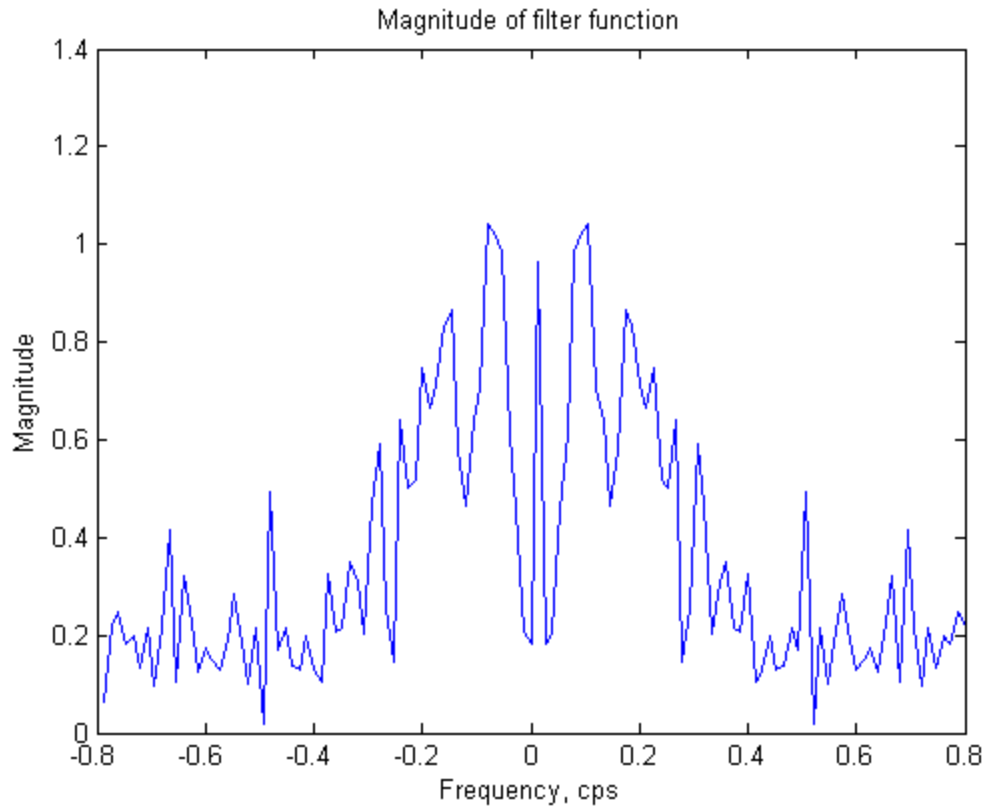
[cross_spec_100_90, freq_bins] = crossspectrum(...
    velocity_90, velocity_100, sample_interval, dof);
[ehat_100_real, ehat_100, freq_bins_real, freq_bins, delta_f] = ...
    varspec_est_multidof(velocity_100, sample_interval, dof);

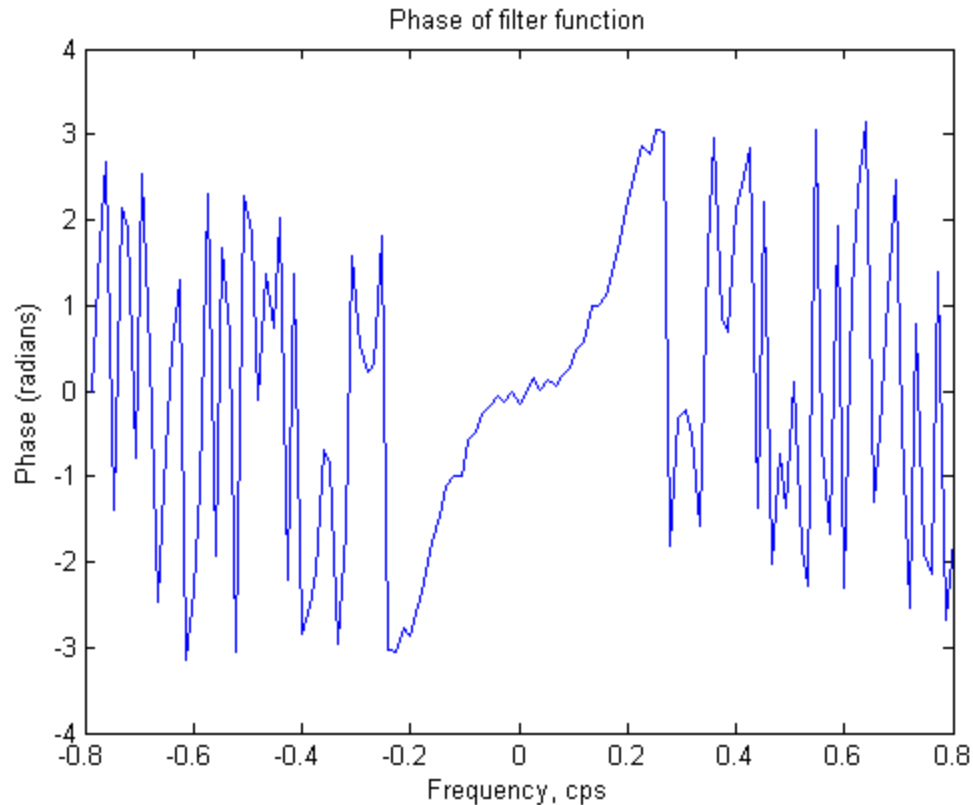
H_100_90 = cross_spec_100_90(1:length(freq_bins)) ./ ehat_100;

mag_h_100_90 = abs(H_100_90);
phase_h_100_90 = angle(H_100_90);

figure()
```

```
plot(freq_bins, fftshift(mag_h_100_90))  
xlabel('Frequency, cps');  
ylabel('Magnitude');  
title('Magnitude of filter function');  
  
figure()  
plot(freq_bins, fftshift(phase_h_100_90));  
xlabel('Frequency, cps');  
ylabel('Phase (radians)');  
title('Phase of filter function');
```





Step 2

You will probably have broken the original data series into 25 segments to calculate $H(!)$. So go back through the 25 segments, Fourier transform the velocity data at range “90” (the forcing input), multiply by the 50 dof $H(!)$, and then inverse-transform to get the predicted record as a function of time at range “80”. Hint: Take care to treat the negative frequencies of the input data and of $H(!)$ correctly, so that your prediction is real. The inverse-transform output should have a very small imaginary part if this is done correctly.

```
% Testing:
%H_100_90 = ones(1,length(H_100_90));

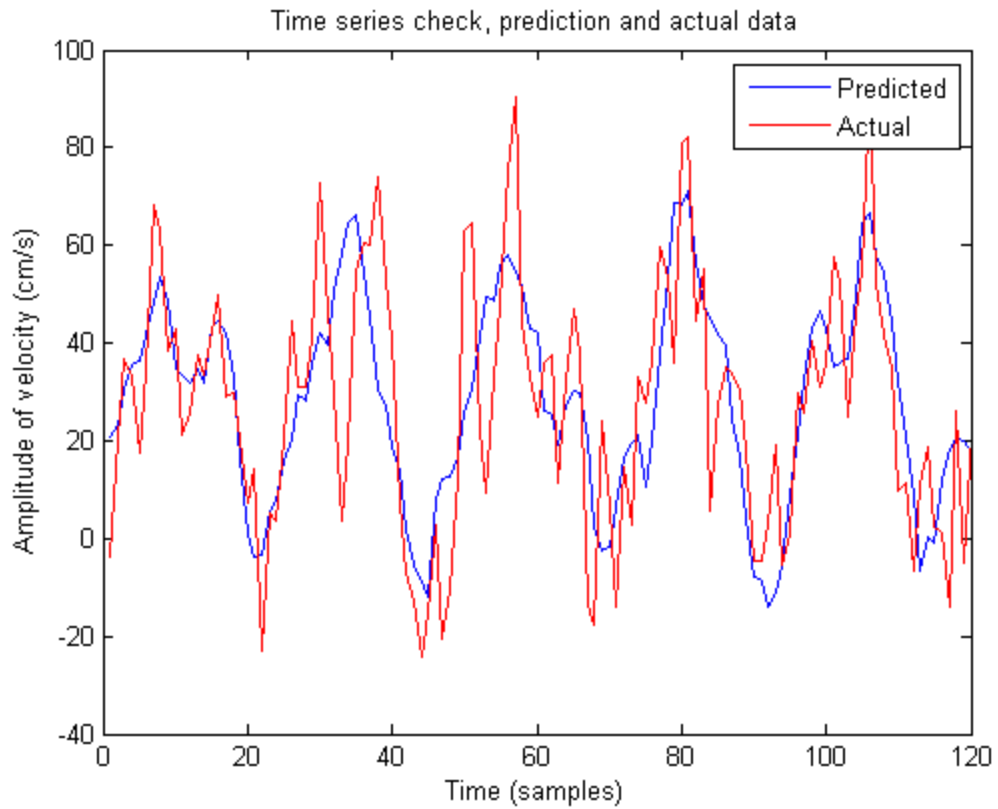
% Get the 25 segments
number_of_sets = 25;
set_size = floor(length(velocity_90) / number_of_sets);
for i=1:number_of_sets
    start_idx = (i-1) * set_size + 1;
    end_idx = start_idx + set_size - 1;

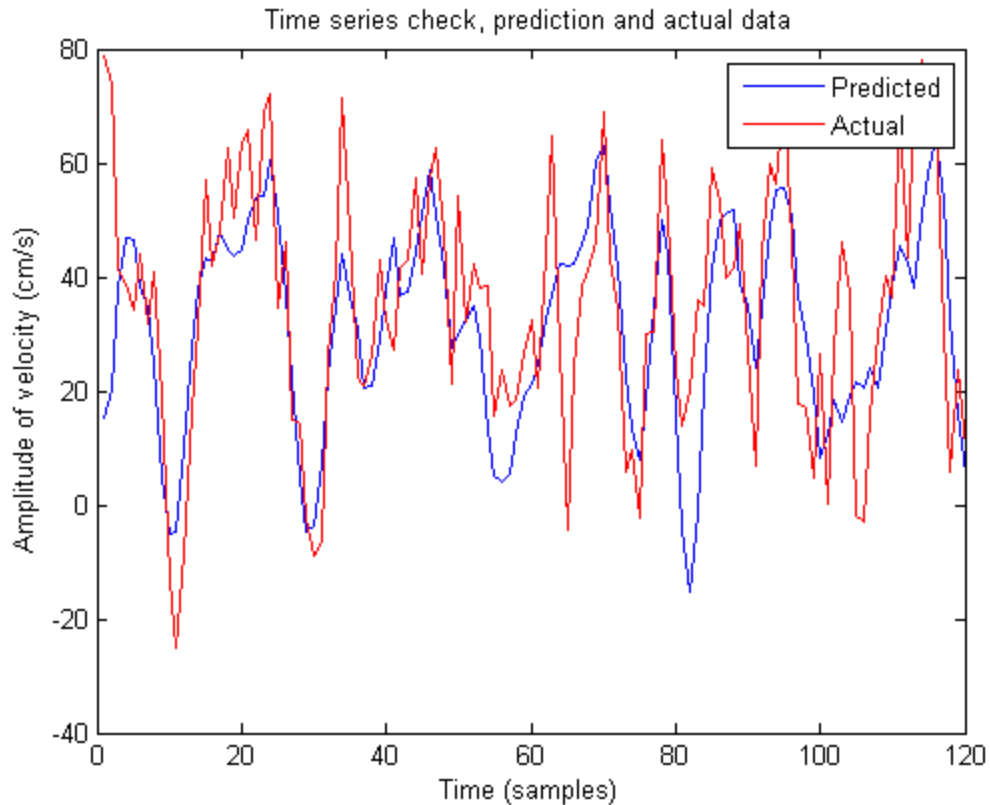
    data = velocity_90(start_idx:end_idx);
    a_90(:,i) = fft(data');
    a_predicted_80 = a_90(:,i)' .* H_100_90;

    predicted_80(:,i) = ifft(a_predicted_80');

    error(:,i) = predicted_80(:,i) - velocity_80(start_idx:end_idx);
```

```
if ((i==5) || (i==20))  
    figure();  
    plot(1:length(predicted_80(:,i)), predicted_80(:,i))  
    hold on;  
    plot(velocity_80(start_idx:end_idx), 'r')  
    hold off;  
    legend('Predicted', 'Actual');  
    xlabel('Time (samples)');  
    ylabel('Amplitude of velocity (cm/s)');  
    title('Time series check, prediction and actual data');  
end  
  
end
```





Step 3

Quantify the accuracy of the prediction by forming time series of $\text{Error}(t) = \text{predicted}(t) - \text{velb4}(80, t)$ in each of the 25 segments. Form power spectra of $\text{Error}(!)$ and $\text{velb4}(80, !)$ at 50 dof. Plot log-log on the same plot. (This is the noise-to-signal ratio) Also, for two of your sub-records, plot $\text{velb4}(80, t)$ and $\text{predicted}(t)$ on the same plot. Which aspects of the wavefield are well-predicted which are not?

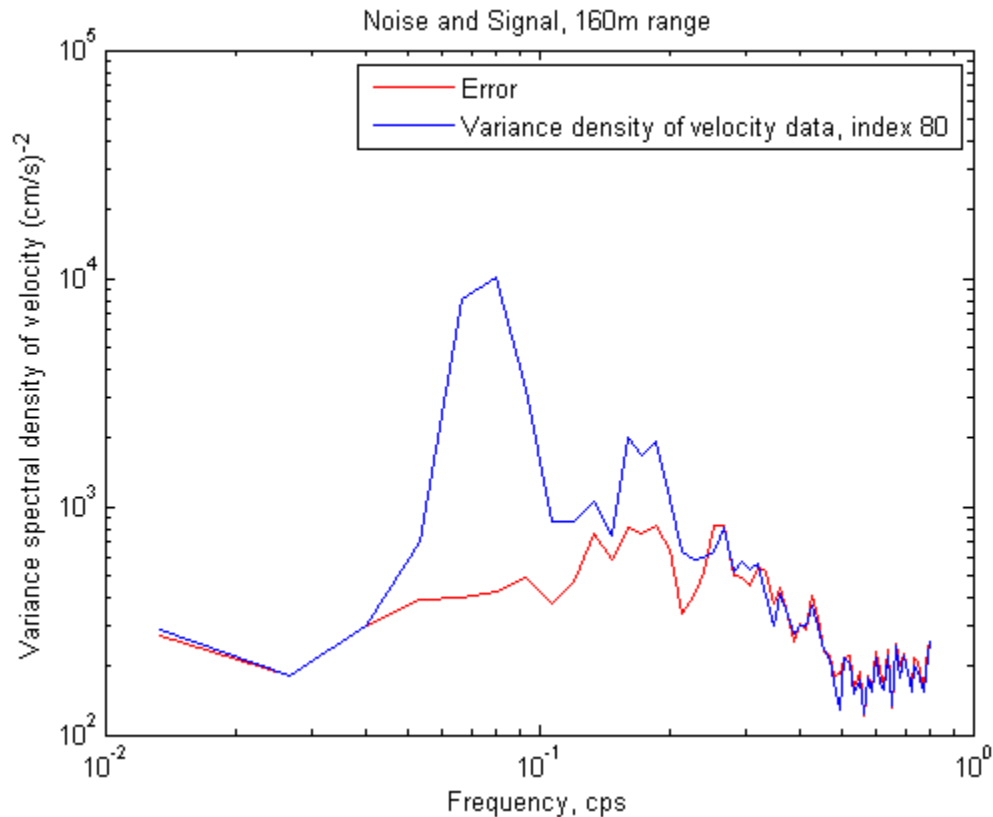
```
error = reshape(error, [], 1,1);
[ehat_error_real, ehat_error, error_freq_bins_real, error_freq_bins, error_delta_f, ...
 varspec_est_multidof(error, sample_interval, dof)];

[ehat_80_real, ehat_80, freq_bins_real, freq_bins, delta_f] = ...
    varspec_est_multidof(velocity_80, sample_interval, dof);

figure();
loglog(freq_bins_real, ehat_error_real, 'r');
hold on;
loglog(freq_bins_real, ehat_80_real, 'b');
legend('Error', 'Variance density of velocity data, index 80');
xlabel('Frequency, cps');
ylabel('Variance spectral density of velocity (cm/s) ^-2');
```

```
title('Noise and Signal, 160m range');  
  
disp('The wavefield is well-predicted in areas where there are ');  
disp('high-magnitude waves (swell, wind peaks).');
```

*The wavefield is well-predicted in areas where there are
high-magnitude waves (swell, wind peaks).*



end

```
function [cross_spec, freq_bins] = crossspectrum(data_a, data_b, sample_interval,  
    % We get 2 DOF from each set of data (thanks to real/imag components.  
    num_sets = ceil(dof / 2);  
  
    len_complete = size(data_a, 1);  
    points_per = floor(len_complete / num_sets);  
  
    delta_f = (1/sample_interval) / points_per; % same as fs / N  
    freq_bins = (points_per/2 * delta_f + delta_f):delta_f:(points_per/2 * delta_f  
  
    % Get the variance spectral density for each subset.  
    for i=1:num_sets  
        start_idx = (i-1) * points_per + 1;  
        end_idx = start_idx + points_per - 1;  
  
        a_a(:,i) = fft(data_a(start_idx:end_idx));
```

```
a_b(:,i) = fft(data_b(start_idx:end_idx));

crosses(i,:) = bsxfun(@times, conj(a_a(:, i)), a_b(:,i));
end

% normalize FFT by n^2
% Get the average values (average of columns)
cross_spec = mean(crosses) ./ delta_f ./ points_per^2;
%cross_spec = reshape(cross_spec, [], 1, 1);

% get rid of the first (mean) value.

end
```

Variance Spectral Density Functions

Take the given data and provide a variance density estimate The freq_bins array will be in reciprocal units to the units of sample_interval. ehat has units of (data_units^2) per (1/sample_interval_units)

```
function [ehat_real, ehat, freq_bins_real, freq_bins, delta_f] = ...
    varspec_est_real(data, sample_interval)

% First, do the FFT to get an array of Fourier coefficients
a = fft(data);

% Figure out our frequency bins.
N = length(data);
delta_f = (1/sample_interval) / N; % same as fs / N
freq_bins_real = 0:delta_f:(N/2 * delta_f);
freq_bins = (-N/2 * delta_f + delta_f):delta_f:(N/2 * delta_f);
%freq_bins = 0:delta_f:N*delta_f - delta_f;

% We are dealing with a real signal, so we only care about positive, real
% frequency components. To get the power, we want to "fold" both sides of
% the spectrum together. Since they are the same, we can just take one
% side and multiply by 2.
% To preserve variance, we have to normalize this result by dividing by
% N^2, due to the Matlab implementation of fft.
ehat_real = 2 * abs(a(1:length(freq_bins_real))).^2 ./ delta_f ./ ...
    (N^2);

% Also, figure out the full spectrum
ehat = abs(a(1:length(freq_bins))).^2 ./ delta_f ./ (N^2);

% The first frequency bin will hold the mean, not a component of the
% variance. Since we are interested in the variance spectrum, drop
% the first bin.
%ehat = ehat(2:end);
ehat_real = ehat_real(2:end);
freq_bins_real = freq_bins_real(2:length(ehat_real)+1);

end
```

```
function [ehat_real, ehat, freq_bins_real, freq_bins, delta_f] = ...
    varspec_est_avg(data, sample_interval, set_size, number_of_sets)

    % Get the variance spectral density for each subset.
    for i=1:number_of_sets
        start_idx = (i-1) * set_size + 1;
        end_idx = start_idx + set_size - 1;

        [ehat_parts_real(i, :), ehat_parts(i, :), freq_bins_real, freq_bins, delta_f] = ...
            varspec_est_real(data(start_idx:end_idx), sample_interval);
    end

    % form the average
    % Get the average values (average of columns)
    ehat = mean(ehat_parts);
    ehat_real = mean(ehat_parts_real);

end

function [ehat_real, ehat, freq_bins_real, freq_bins, delta_f] = ...
    varspec_est_multidof(data, sample_interval, dof)

    % We get 2 DOF from each set of data (thanks to real/imag components).
    num_sets = ceil(dof / 2);

    len_complete = size(data, 1);
    points_per = floor(len_complete / num_sets);

    [ehat_real, ehat, freq_bins_real, freq_bins, delta_f] = ...
        varspec_est_avg(data, sample_interval, points_per, num_sets);

end
```

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