
SIO221A Homework #5 (Eric Gallimore)

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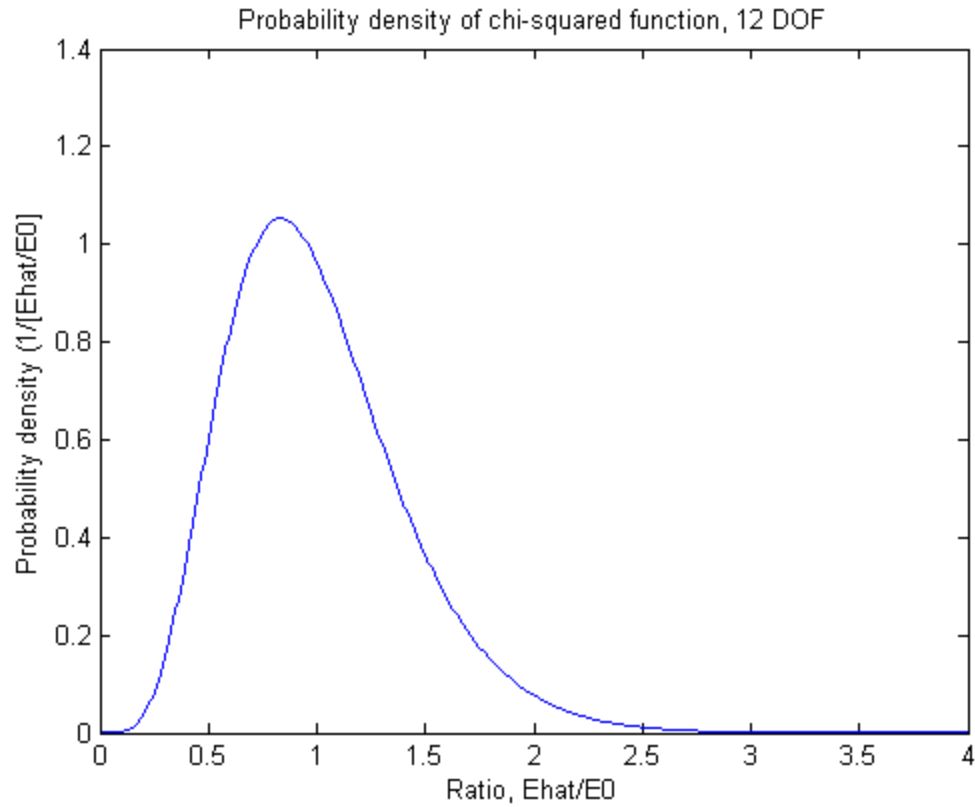
1: Plot PDF of \hat{E}/E_0 for 12 DOF

```
%clear all
close all

ratios = 0:0.01:4;
N = 12;

for i=1:length(ratios);
    ratio = ratios(i);
    ps(i) = N * (2^(N/2)*gamma(N/2))^-1 * (N*ratio).^(N/2 - 1) * ...
        exp(-N/2 * ratio);
end

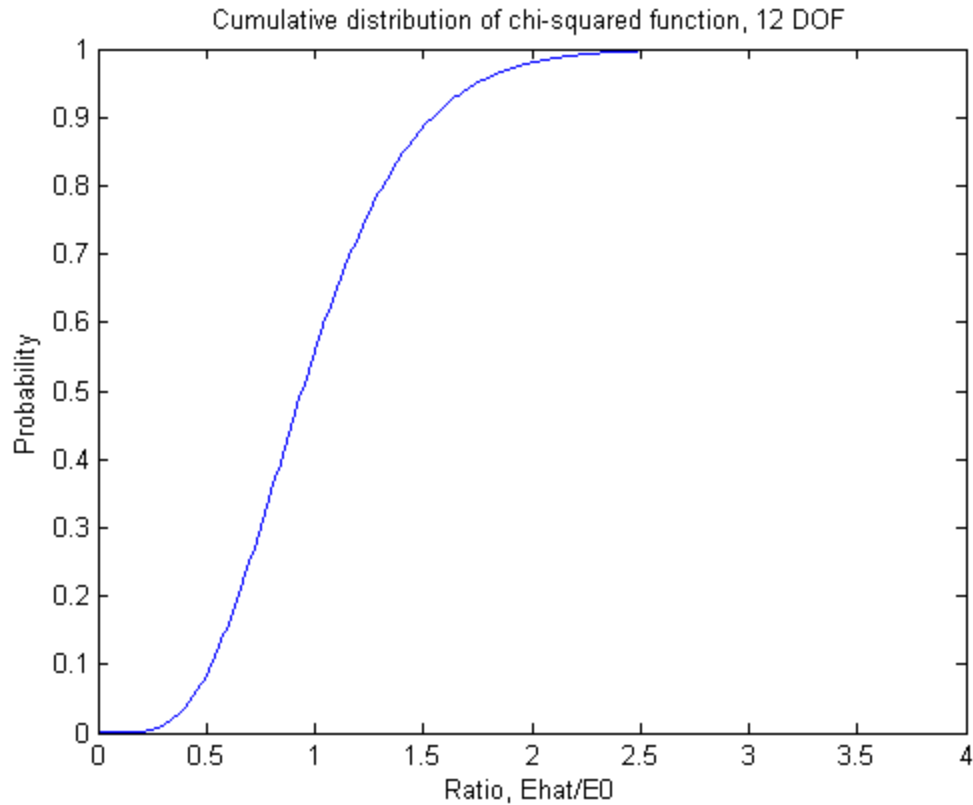
figure()
plot(ratios, ps)
title('Probability density of chi-squared function, 12 DOF')
ylabel('Probability density (1/[Ehat/E0])');
xlabel('Ratio, Ehat/E0');
```



Plot the associated CDF

Find the CDF... just integrate under the curve.

```
cdf = cumsum((ps.*0.01));  
  
figure()  
plot(ratios, cdf);  
title('Cumulative distribution of chi-squared function, 12 DOF')  
ylabel('Probability');  
xlabel('Ratio, Ehat/E0');
```

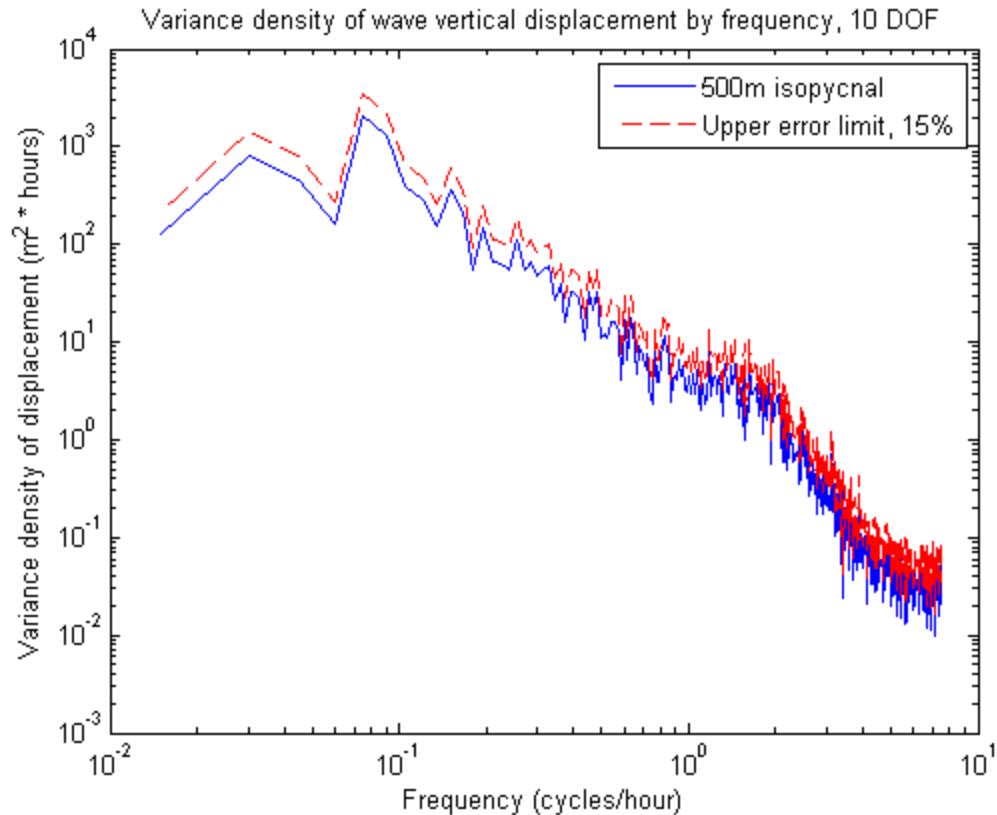


Find an upper error bound

Suppose we want to define an upper error bound for our spectral estimate, such that there is only a 15% chance that the true process spectrum is greater than that bound. Identify the corresponding quantile of \hat{E}/E_0 . Plot the appropriate bound at all frequencies on the spectral estimate.

```
i_15 = find(cdf > 0.15, 1);
ratio_15 = ratios(i_15);

figure()
loglog(freq_bins_500m_average, Ehat_500m_average);
hold on;
loglog(freq_bins_500m_average, Ehat_500m_average ./ ratio_15, 'r--');
ylabel('Variance density of displacement (m^2 * hours)');
xlabel('Frequency (cycles/hour)');
legend('500m isopycnal', 'Upper error limit, 15%');
title('Variance density of wave vertical displacement by frequency, 10 DOF');
```



4) Suppose, for that same error bound, you wanted to be 95% certain that

E_0 is less than the upper bound. How much longer a data record would you need than the one you have, if you want to maintain the same frequency resolution that the 12 DOF estimate now has?

% If we wanted to have a 95% certainty for the same error bounds, we would
% need the cdf of E_{hat} / E_0 to be 0.05 at 0.59.

% We could solve this analytically, but doing it numerically is easy.

```
for N = 12:2:48
    clear('ps');
    for i=1:length(ratios)
        ratio = ratios(i);
        ps(i) = N * (2^(N/2)*gamma(N/2))^(-1) * (N*ratio).^(N/2 - 1) * ...
            exp(-N/2 * ratio);
    end
    cdf = cumsum((ps.*0.01));
    i_05 = find(cdf > 0.05, 1);
    ratio_05 = ratios(i_05);
    if ratio_05 > ratio_15
        fprintf('0.95 certainty occurs at N = %d\n', N);
        break;
    end
end
```

0.95 certainty occurs at $N = 28$

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